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Working Paper

DT/180/2018

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This paper has developed the theoretical background of Okun's Law (LO) and an integrated IS-LM-LO model to better understand the effects of economic policies on unemployment. As stated and demonstrated, the Okun's Law Theorem confirms the existence of a negative relationship between the rate of economic growth and the changes in the unemployment rate. However, this Okun's relationship cannot be considered stable. Among other things, shocks on labor force, wages and gross profits induce structural changes in Okun's relationship. The structural change in Okun's relationship can be virtuous or vicious. When the economic dynamic begets a vicious structural change in Okun's relationship in such a way the new Okun threshold that it induces is always greater than the observed growth rate, the unemployment rate increases. Economic growth has a greater impact on unemployment when it is strong and begets a virtuous structural change in Okun's relationship. Thus, from the analysis of the IS-LM-LO model, it appears that fiscal and / or monetary policy, although having a positive effect on economic growth, would lead to an increase in unemployment if it generates a vicious structural change in the Okun's relationship. The most effective economic policy in the fight against unemployment is one that, in addition to accelerating growth, induces a virtuous structural change in Okun's relationship.

Résumé : Un théorème pour la Loi d'Okun

Ce papier a développé les fondements théoriques de la loi d'Okun (LO) et un modèle intégré IS-LM-LO permettant de mieux cerner les effets de politiques économiques sur le chômage. Enoncé et démontré, le théorème de la loi d'Okun confirme l'existence d'une relation négative entre le taux de croissance économique et la variation du taux de chômage. Cependant, cette relation d'Okun ne peut pas être considérée comme stable. Entre autres, les chocs sur la main-d'œuvre, les salaires et les profits bruts induisent des changements structurels dans la relation d'Okun. Le changement structurel dans la relation d'Okun peut être vertueux ou vicieux. Lorsque la dynamique économique engendre un changement structurel vicieux dans la relation d'Okun de sorte que le nouveau seuil de croissance qu'elle induit est toujours supérieur au taux de croissance observé, le taux de chômage augmente. La croissance économique a un impact plus important sur le chômage lorsqu'elle est forte et engendre un changement structurel vertueux dans la relation d'Okun. Ainsi, de l'analyse du modèle IS-LM-LO, il ressort que la politique budgétaire et/ou monétaire, bien qu'ayant un effet positif sur la croissance économique, entrainerait une hausse du chômage si elle engendre un changement structurel vicieux dans la relation d'Okun. La politique de relance économique la plus efficace dans la lutte contre le chômage est celle qui, en plus d'accélérer la croissance, induit un changement structurel vertueux dans la relation d'Okun.

Keywords: Okun's Law Theorem, Unemployment, Economic Growth, IS-LM-LO Model
Mots clés : Théorème de la Loi d'Okun, chômage, croissance économique, modèle IS-LM-LO

JEL classification: E24, E63.

[1] My sincere acknowledges to all those who made this article possible, especially to Messrs Hien B. Léon, Fayama Karfa and Savadogo Lamine. My deepest gratitude to Emeritus Professor Jean-Pierre Lachaud for his constant support.

Summary

1. Introduction

Economists have always observed the relationship between economic growth and unemployment. However, Arthur Okun (1962) has been the first to formally express the existence of a negative relationship between growth and unemployment by establishing an empirical relationship from US statistical data. Since then, numerous empirical studies have been carried out to verify the robustness of Okun's law, as well as the presence of structural changes in this empirical relationship (Gaëtan, 2014). These empirical works seem to have settled the question of the robustness of Okun's law. However, they did not exhaust the question about the variations of the coefficient of Okun and especially the causes of these variations.

Moreover, one of the weaknesses of Okun's law is related to the fact that its theoretical foundations have not yet been built up. In our humble opinion, a law as important for macroeconomics deserves to have theoretical foundations in order to definitively confirm its robustness and to answer questions about the presence and causes of structural changes in the empirical relationship. In addition, knowledge of the theoretical foundations of Okun's law would complement the IS-LM model to make it a real tool for analyzing the impact of economic policies on underemployment, shedding light on how the increase in national income Y (due to the shifts in the IS and LM curves) involves the reduction of underemployment.

So, this paper has two objectives. The first objective is to find out the theoretical background of Okun's law in order to better understand the relationship between economic growth and unemployment changes. The second objective is to add Okun law (LO) in the IS-LM model as a tool for analyzing the impact of economic policies on underemployment. So, this paper is structured in three sections. The first section gives a brief overview of Okun's law; the second section develops the theoretical background of Okun's law on the basis of undebatable economic principles; the third section is dedicated to the design of the integrated model for analyzing the effects of economic policies on unemployment.

2. Outline of the debate on Okun's Law and the backgrounds of our theoretical approach

In 1962, American economist Arthur M. Okun highlighted a negative empirical relationship between unemployment changes and economic growth based on US data for the period 1947-1960. The most famous form of Okun's relationship is given by the equation (1) below. In this equation, Y is the output (Gross Domestic Product (GDP)) and g* is the growth rate of potential output. β is the Okun coefficient; it measures the elasticity of the unemployment rate out of the output. The Okun coefficient being negative, g* is the economic growth threshold above which the unemployment rate decreases. When the growth rate is lower than g*, the unemployment rate increases.

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$$\Delta U = \beta\left(\frac{\Delta Y}{Y} - g^*\right) \qquad (1)$$

Okun's empirical work showed in 1962 an economic growth of 3% led to a 1% reduction in the unemployment rate related to their trends. Since then, several empirical studies have been conducted on measuring the elasticity of unemployment changes in relationship to economic growth. So, Okun's law has become a core element in macroeconomics with many estimations of the value of the Okun coefficient. But, nowadays, there is no consensus on the value and interpretation of this coefficient. In a sample of 28 articles published between 1989 and 2009 on Okun's law, Gaëtan (2014) counted 269 Okun coefficients ranging from -3.22 to 0.17.

The lack of consensus is enabled by the fact that the theoretical foundations of Okun's law are not yet known. Indeed, the absence of theoretical foundations has arisen several controversies in the Okun coefficient estimation method. According to Gaëtan (2014), the controversy elements in the empirical strategies used to estimate Okun's relationship concern (i) the choice of output or unemployment as endogenous variable, (ii) the use of a static or dynamic specification, (iii) the choice of the frequency of the data, (iv) the choice between a "first differences" version or a "gap" version, (v) the measurement of potential output and the natural unemployment rate.

In addition, the literature is fraught with many differences in the interpretation of the changes in Okun's relationship. Prachowny (1993) wrote: "Whereas Okun interpreted his coefficient as *mutatis mutandis*, most current applications imply *ceteris paribus.* Many textbooks in macroeconomics merely point to the relatively stable relationship between changes in unemployment rates and changes in output without mentioning other factors that influence latter." For example, while Owyang and Sekhposyan (2012) point out that recent recessions in the United States tend to significantly modify the value of the Okun coefficient, Ball and al. (2013) support the stability of Okun's relationship. At the same time, Marra and Zanin (2012) emphasize that the Okun coefficient has increased in the euro zone as a result of labor market reforms. A decade before, Sögner and Stiassny (2002) linked the value of the Okun coefficient to the degree of flexibility of the labor market. While for Knotek (2007) and Beaton (2010), the variations in the Okun coefficient are rather linked to the expansion and recession phases of the US economy.

Most of the debate about Okun's law is led through empirical studies that consider the Okun coefficient as a mere econometric parameter and not as a function of economic and/or demographic factors. However, such a coefficient can be better understood only if, theoretically, we know variables that influence it. In the literature, the theoretical approaches of Okun's law are based on specifics production functions. Prachowny (1993) derives the relationship between unemployment changes and output from production function for the economy with, of course, many controversial hypotheses. For example, this author used particular form of Cobb-Douglass' production function and assumed that actual and optimal levels of the capital input and the disembodied technology factor are respectively equal. One of the drawback of all these theoretical approaches is that they cannot apprehend the exact expression of Okun coefficient.

Given its "universal" character, if it exists, Okun's law must not depend on any hypothesis. The theoretical foundations of Okun's law must be constructed from undisputed basic principles of economics and not on the basis of particular hypotheses. So, the following section is dedicated to the design of the theoretical foundations of Okun's law, presented as a theorem. Two universal equalities, valid in any free enterprise economy, form the basis of the reasoning of the theorems which are stated and demonstrated. The first equality (equation 2) is the accounting identity of GDP in terms of production. It states that real GDP (at market price) is equal to average real wages multiplied by total employment (w*L), plus the real value of the Gross Operating Surplus (i.e. real gross profits, noted π), as well as the real net duties and taxes on goods and services (DT). The second equality (equation 3) is based on the definition of the labor force (P), i.e. the total number of persons employed in a job (L), plus the total number of unemployed (U*P); U being the unemployment rate.

$$Y = wL + \pi + DT \qquad (2)$$

$$P = L + UP \qquad (3)$$

3. Theoretical Foundations of Okun's Law

The link between economic growth and unemployment depends, on the one hand, on the link between the dynamics of job creation and economic growth, and on the other hand, on the gap between the dynamics of job creation and demographic dynamics. Therefore, we will proceed in two steps to design the foundations of Okun's law. The first one is to set up the relationship between economic growth and job creation and then use it to develop the relationship between economic growth and unemployment changes, i.e. Okun's relationship.

1. Relationship between economic growth and employment

From equation (2), without any assumptions, we can notice that the link between the growth rate of real GDP (Y) and that of employment (L) will depend on the respective growth rates of the average real wage (w) and real gross profit (π), as well as respective shares in real GDP of real gross profit, real payroll, and net duties and taxes on goods and services. This is what the theorem tells us below.

a. Statement of the "growth-employment" theorem

Say any free enterprise economy considered at two dates t_0 and t_1.
Let's note:
- L : total employment ;
- Y : real GDP (at market price) ;
- θ : the rate of real gross profit increase between t_0 and t_1;
- ρ : the share of real gross profit in real GDP at t_0;
- d : the rate of increase in real net duties and taxes on goods and services between t_0 and t_1;
- σ : the share of net duties and taxes on goods and services in real GDP at t_0;
- α : the rate of average real wage increase between t_0 and t_1.

Then, the rate of increase of total employment between t_0 and t_1 is given by equation (4):

$$\frac{\Delta L}{L_0} = \frac{1}{(1+\alpha)(1-\rho-\sigma)}\left[\frac{\Delta Y}{Y_0} - \left(\rho\theta + (1-\rho-\sigma)\alpha + \sigma d\right)\right] \qquad (4)$$

b. Proof of the "growth-employment" theorem

To prove this theorem, we will start from equation (2) and we will consider two dates t_0 and t_1. So, for each of these dates, real GDP (Y) is equal to the sum of real payroll, gross real profits and import duties and net taxes on goods and services (equations (5) and (6)). In these equations, w, π, and DT respectively denote the average real wage, the gross real profit, and the import duties and net taxes on goods and services.

$$Y_0 = w_0 L_0 + \pi_0 + DT_0 \qquad (5)$$

$$Y_1 = w_1 L_1 + \pi_1 + DT_1 \qquad (6)$$

Let α be the rate of increase of the average real wage between t_0 and t_1, then we have: $w_1 = (1+\alpha)w_0$. By differentiating between equation (6) and equation (5) and putting w_0 in factor, we obtain the equation (7).

$$Y_1 - Y_0 = w_0\left[(1+\alpha)L_1 - L_0\right] + \pi_1 - \pi_0 + DT_1 - DT_0 \qquad (7)$$

This equation (7) is equivalent to equation (8) below. To be convinced of this, we must add the term null $w_0\alpha L_0 - w_0\alpha L_0$ to the equation (7) and make an appropriate arrangement.

$$Y_1 - Y_0 = w_0\left[(1+\alpha)(L_1 - L_0) + \alpha L_0\right] + \pi_1 - \pi_0 + DT_1 - DT_0 \tag{8}$$

By dividing equation (8) by w_0L_0, i.e. the payroll at t_0, we get the equation (9).

$$\frac{Y_1 - Y_0}{w_0 L_0} = \left[(1+\alpha)\frac{L_1 - L_0}{L_0} + \alpha\right] + \frac{\pi_1 - \pi_0}{w_0 L_0} + \frac{DT_1 - DT_0}{w_0 L_0} \tag{9}$$

Knowing that σ is the share of import duties and net taxes on goods and services (DT) in real GDP at t_0 and ρ the share of real gross profits in real GDP at t_0, we obtain the following equality $w_0L_0 = (1-\rho-\sigma)\, Y_0$ from the equation (5). Then, the equation (9) gives the equation (10).

$$\frac{Y_1 - Y_0}{(1-\rho-\sigma)Y_0} = \left[(1+\alpha)\frac{L_1 - L_0}{L_0} + \alpha\right] + \frac{\pi_1 - \pi_0}{(1-\rho-\sigma)Y_0} + \frac{DT_1 - DT_0}{(1-\rho-\sigma)Y_0} \tag{10}$$

The equation (10) is equivalent to the equation (11) which is obtained by multiplying the last two terms of the right-hand side of the equation (10) respectively by (π_0/π_0) and by DT_0/DT_0.

$$\frac{Y_1 - Y_0}{(1-\rho-\sigma)Y_0} = \left[(1+\alpha)\frac{L_1 - L_0}{L_0} + \alpha\right] + \frac{\pi_1 - \pi_0}{\pi_0}\frac{\pi_0}{(1-\rho-\sigma)Y_0} + \frac{DT_1 - DT_0}{DT_0}\frac{DT_0}{(1-\rho-\sigma)Y_0} \tag{11}$$

By multiplying the equation (11) by (1-ρ-σ) and simplifying, we obtain the equation (12).

$$\frac{Y_1 - Y_0}{Y_0} = (1-\rho-\sigma)\left[(1+\alpha)\frac{L_1 - L_0}{L_0} + \alpha\right] + \frac{\pi_1 - \pi_0}{\pi_0}\rho + \frac{DT_1 - DT_0}{DT_0}\sigma \tag{12}$$

By replacing, in equation (12), the respective growth rates of the real gross profits and the DT by their respective values θ and d, and by expressing the rate of increase in employment according to the real GDP growth rate, we obtain the equation (13) which is the result of the theorem.

$$\frac{L_1 - L_0}{L_0} = \frac{1}{(1+\alpha)(1-\rho-\sigma)}\left[\frac{Y_1 - Y_0}{Y_0} - \left(\alpha(1-\rho-\sigma) + \rho\theta + \sigma d\right)\right] \tag{13}$$

c. Lessons from the growth-employment theorem

Firstly, the theorem confirms the idea according to which any economic growth is not equal to an increase in employment. For a given structure of income distribution between profit, wages, and net duties and taxes on goods and services at time t_0, economic growth between t_0 and t_1 will be accompanied by an increase only if it is characterized by low rates of increase in average real wage (α), real gross profit (θ), and net duties and taxes (d), so that real GDP growth rate be greater than g (α, θ, d) given by equation (14). Conversely, when the growth rate of real GDP is less than g (α, θ, d), economic growth yields a reduction in employment.

$$g(\alpha, \theta, d) = \rho\theta + (1-\rho-\sigma)\alpha + \sigma d \tag{14}$$

Secondly, the theorem confirms that the capacity of economic growth to create employment is not just a matter of the threshold of growth rate to be exceeded; it also depends on the nature or quality of economic growth. Indeed, as the equation (14) shows, g (α, θ, d) is a variable threshold. It represents the rate of economic growth without net job creation between t_0 and t_1, or the share of economic growth that would benefit to insiders (workers at t_0), to employers and to the state budget, at the expense of the outsiders' entry (unemployed at t_0) into the labor market.

So, all else being equal, economic growth is more conducive to job creation when it is characterized by moderate increases in real gross profits, average real wages, import duties and net taxes on real wages on goods and services. Economic growth characterized by relatively moderate increases in real gross profits and average real wages reflects the fact that employers responded to the increase in activity more by hiring new workers than by other means of growth of production such as increased working time, the use of low-labor-intensive technologies, the replacement of low-skilled labor by more skilled labor. On the other hand, when employers react to the increase of their activities more by internal processes of increase of production and/or by dismissals of certain categories of workers, the economic growth yielded a strong growth of the wage real average and / or real gross profit and, thus, by a threshold g (α, θ, d) higher than the economic growth rate.

Without claiming to have exhausted all the lessons that this first theorem can provide, we used in the following subsection to prove the theorem that confirms and specifies Okun's law that we decide to call "Okun's Law Theorem".

2. Okun's Law Theorem

In an economy, the creation of new jobs is not equal to a decrease of the unemployment rate. Indeed, unemployment depends on both the ability of the economy to offer new job opportunities and the dynamics of the labor force. So, the theorem of Okun's law takes into account these two dimensions.

a. Statement of Okun's Law Theorem

Say any free enterprise economy considered at two dates t_0 and t_1.
Let's note:

- Y: real GDP (at market price);
- P: total of labor force;
- λ : the rate of labor force increase between t_0 and t_1;
- c_0 : the capacity of real GDP to pay the entire labor force at the average wage at t_0, ($c_0 = Y_0/w_0P_0$) ;
- α : the rate of average real wage (w) increase between t_0 and t_1;
- θ : the rate of real gross profit increase between t_0 and t_1;
- ρ : the share of real gross profit in real GDP at t_0 ;
- σ : the share of net duties and taxes on goods and services in real GDP at t_0 ;
- d : the rate of increase in real net duties and taxes on goods and services between t_0 and t_1.

So, the changes of the unemployment rate (ΔU) between t_0 and t_1 are given by equation (15) below:

$$\begin{cases} \Delta U = -\dfrac{c_0}{(1+\lambda)(1+\alpha)}\left[g(Y) - g^*\right] \\ with: \\ g(Y) = \dfrac{\Delta Y}{Y_0}\ ;\ (i.e.\ rate\ of\ real\ GDP\ increase between_0\ and\ t_1) \\ g^* = \rho\theta + (1-\rho-\sigma)(\alpha+\lambda+\alpha\lambda) + \sigma d\ ;\ (i.e.\ Okun\ threshold) \end{cases} \tag{15}$$

b. Proof of Okun's Law Theorem

To prove this second theorem, consider equation (3) which expresses the definition of the labor force, i.e. $P = L + UP$, with P, L, U being respectively the labor force, total employment and the unemployment rate. So, on dates t_0 and t_1, we have equations (16) and (17).

$$L_0 = (1-U_0)P_0 \tag{16}$$

$$L_1 = (1-U_1)P_1 \tag{17}$$

By differentiating between the two previous equations, we obtain the equation (18).

$$L_1 - L_0 = (1-U_1)P_1 - (1-U_0)P_0 \tag{18}$$

By developing the equation (18) and adding the null term U_0P_1-U_0P_1, then by operating proper arrangements, we find the equation (19).

$$L_1 - L_0 = (1-U_0)(P_1 - P_0) - (U_1 - U_0)P_1 \tag{19}$$

By dividing the equation (19) by L_0 and observing that $P_1=(1+(P_1-P_0)/P_0)*P_0$, we get the equation (20).

$$\frac{L_1 - L_0}{L_0} = (1-U_0)\frac{P_0}{L_0}\frac{(P_1 - P_0)}{P_0} - (U_1 - U_0)(1+\frac{P_1 - P_0}{P_0})\frac{P_0}{L_0} \tag{20}$$

Noting λ the rate of labor force increase between t_0 and t_1, and observing that from equation (16) one can write $P_0/L_0 = 1/(1-U_0)$, the equation (20) gives the equation (21) below. This relationship expresses the rate of increase in total employment as a function of the rate of increase in the labor force, the unemployment rate at t_0 and the changes in the unemployment rate between t_0 and t_1.

$$\frac{\Delta L}{L_0} = \lambda - \Delta U \frac{(1+\lambda)}{(1-U_0)} \tag{21}$$

Let us use the first theorem (equation 4) and equalize the two expressions of the rate of increase in the employment given by the equations (4) and (21). This gives the equation (22) below. By drawing the changes in the unemployment rate and making a proper arrangement, we get the equation (23).

$$\lambda - \Delta U \frac{(1+\lambda)}{(1-U_0)} = \frac{1}{(1+\alpha)(1-\rho-\sigma)}\left[\frac{\Delta Y}{Y_0} - \left(\rho\theta + (1-\rho-\sigma)\alpha + \sigma d\right)\right] \tag{22}$$

$$\Delta U = -\frac{1-U_0}{(1+\lambda)(1+\alpha)(1-\rho-\sigma)}\left[\frac{\Delta Y}{Y_0} - \left(\rho\theta + (1-\rho-\sigma)(\alpha+\lambda+\alpha\lambda) + \sigma d\right)\right] \tag{23}$$

Equation (23) reveals not only Okun's law, but also the expression of Okun's coefficient. Note β the Okun coefficient. Then, as expressed in equation (24), the Okun coefficient is a function of the rate of increase in the labor force λ and the rate of increase in the average real wage α.

$$\beta(\lambda\,;\alpha) = -\ \frac{1-U_0}{(1+\lambda)(1+\alpha)(1-\rho-\sigma)} \tag{24}$$

Knowing by definition that: $(1-\rho-\sigma) = w_0L_0/Y_0$; $(1-U_0) = L_0/P_0$ and $c_0 = Y_0/w_0P_0$, the equation (24) of the Okun coefficient can then be written in the form of the equation (25) below.

$$\beta(\lambda\,;\alpha) = -\ \frac{c_0}{(1+\lambda)(1+\alpha)} \qquad (25)$$

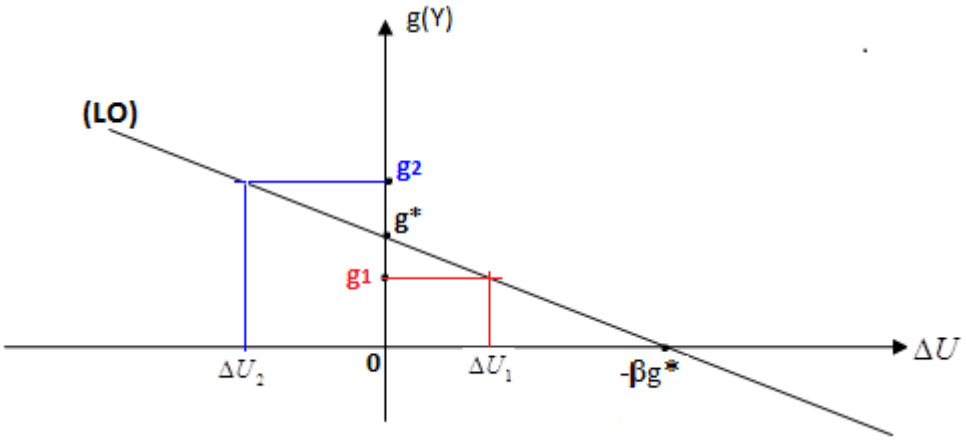
From equations (23) and (25), we deduct at last the equation (26), i.e. the result of the theorem.

$$\Delta U = -\frac{c_0}{(1+\lambda)(1+\alpha)}\left[\frac{\Delta Y}{Y_0} - \left(\rho\theta + (1-\rho-\sigma)(\alpha+\lambda+\alpha\lambda) + \sigma d\right)\right] \qquad (26)$$

3. Structural changes in Okun's relationship

The theorem confirms the existence of Okun's law in any free market economy. That is to say, there is a negative relationship between the change in the unemployment rate and the real GDP growth rate in any free enterprise economy. This negative relationship is illustrated in Graph 1 where the line (LO) represents the Okun relationship. It cuts the axis representing the economic growth rate in g* and the axis representing the changes of the unemployment rate in $\Delta U(0) = -\beta g^*$.

Graph 1: Graphical Illustration of Okun's Law (LO)



Source: This paper

Graph 1 shows that for a given Okun's relationship, the change in the unemployment rate is positive when the real GDP growth rate is less than g*; whereas the change in the unemployment rate is negative when the growth rate of real GDP is greater than g*. Thus, for a given Okun relationship, there is a threshold g* of the GDP growth rate below which the unemployment rate increases.

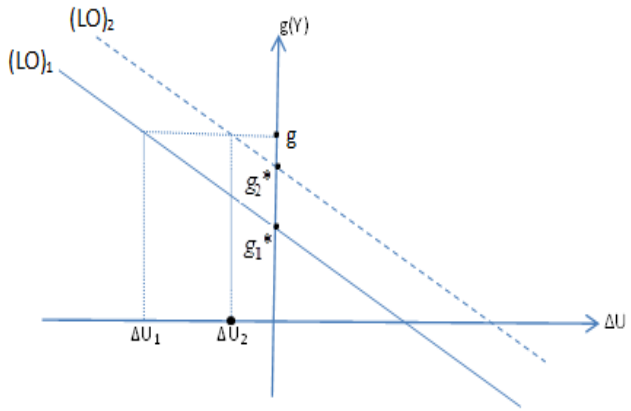
In addition, the theorem shows that the Okun relationship is characterized by the coefficient β and the threshold g* which are functions of demographic and economic variables. Therefore, it cannot be considered stable in time and space.

For a period t_0 and t_1, the Okun β coefficient (equation 25) is a function of the overall wage capacity at the beginning of the period, the rate of increase of the average real wage and the rate of increase of the average wage of active population. The absolute value of the Okun coefficient is decreasing with the growth rates of the labor force and the average real wage. This indicates that, all things being equal, the higher the dynamics of the labor force and / or the average real wage, the lower the elasticity of the change in the unemployment rate to the rate of real GDP growth.

The threshold g* increases with the respective rates of increase in real gross profits, average real wages, labor force, real value of import duties and net taxes on goods and services. Also, it can be seen that g* represents the real GDP growth rate that would be required to ensure full employment of labor, given the income distribution structure, and in view of increased profits and average real wages and import duties and net taxes on goods and services.[2] Thus, the more the economic dynamics fuel the increases in real gross profits and the average real wage, the less it creates employment and the higher the threshold g* is relatively high. This means that reducing the unemployment rate requires even higher levels of growth. On the other hand, the less dynamic the economy drives the increase in profits and the average real wage, the more favorable it is to employment, the lower the threshold.

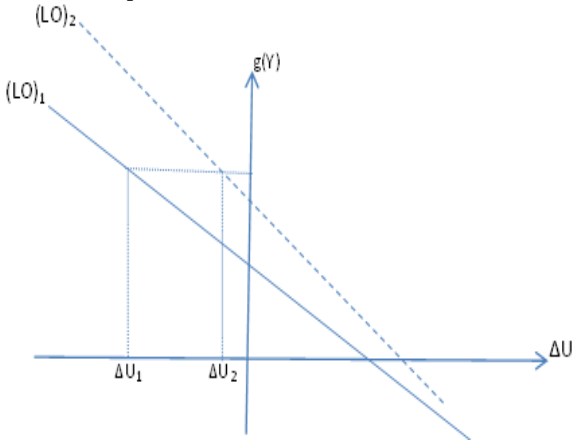
Thus, all economic, demographic and social policies and reforms that affect average real wages, gross real profits, import duties and net taxes on goods and services and / or the labor force result in structural changes in Okun's relationship. These include, but are not limited to, labor market policies, tax policies, investment incentive policies and migration policies. As a result, Okun's relationship depends in particular on the economic, demographic and social policies and reforms of the country and the moment considered. Thus, the stability of Okun's relationship cannot be the rule, as economic and demographic policies and reforms and / or labor market reforms differ across countries and times.

Graph 2: Structural evolution of the Okun relationship when θ or d increases



Source : This Paper

Graph 3: Structural evolution of the Okun relationship when α or λ increases



Source : This Paper.

Graphs 2 and 3 illustrate the structural changes in Okun's relationship as a result of increased rates of increase in real gross profits, average real wages, labor force, import tariffs, and net taxes on goods and services.

When the growth of real gross profits accelerates from θ_1 to θ_2, all things being equal, the curve (LO) of the Okun relationship moves to the right, for example from (LO) $_1$ to (LO) $_2$ as shown in graph 2. More exactly, it undergoes a horizontal translation of $\beta\rho$ (θ_1-θ_2). In this case, for the same level of economic growth g, the fall in unemployment will now be smaller, with a difference equal to $\beta\rho$ (θ_1-θ_2). There is a similar shift in the LO curve due to the acceleration of growth in import duties and net taxes on goods and services.

When the rate of increase in the average wage and / or the rate of increase in the labor force increases, the absolute value of the Okun coefficient decreases while the threshold g* increases. Then, the displacement of the curve (LO) can be decomposed into two phases: (i) it will pivot around its

[2] Indeed, if the economy is in a situation of full employment and that α, θ, λ and d are the respective growth rates of the average real wage (w), real gross profit (π), the labor force (P) and the CTs between t_0 and t_1, the real GDP of full employment would be $Y_0^* = w_0 P_0 + \pi_0 + DT_0$ at t_0 and from $Y_1^* = w_0(1+\alpha)P_0(1+\lambda)+\pi_0(1+\theta)+DT_0(1+d)$ to t_1. Therefore, the real GDP growth rate of full employment of labor between t_0 and t_1 would be equal to g*.

point of intersection with the axis of the growth rate of Y following the decrease of the coefficient of Okun, before (ii) horizontal translation to the right. In this case too, for the same level of economic growth, the decline in unemployment will now be smaller.

4. Integrated Model IS-LM-LO

The lessons from the Okun's Law Theorem call for a re-examination of the IS-LM model as a tool for analyzing the impact of economic policies on unemployment.

1. Brief reminder on the IS-LM model

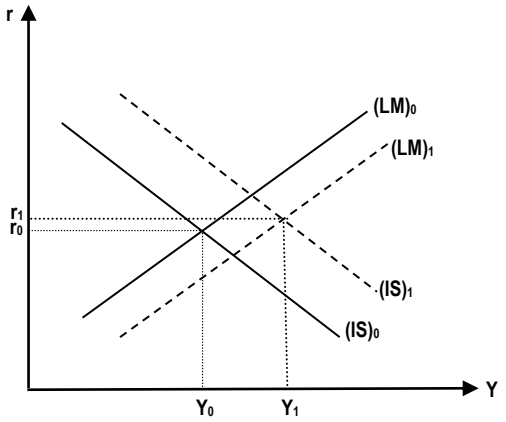
In the 1930s, the United States experienced massive unemployment and a substantial drop in income. This depression challenged the validity of the classical theory which was unable to explain it. In fact, according to classical theory, national income depends on the supply of factors and available technologies; yet neither factor offerings nor technologies had changed significantly during the depression. Then in 1936, the economist John Maynard Keynes proposed a new approach in his book entitled "The general theory of employment, interest and money". According to Keynes, it is the weakness of the effective demand which is the cause of the depression of the incomes and the increase of the unemployment. For Keynes, the economy may be in a state of underemployment equilibrium; in this case, the intervention of the State through the budgetary policy would boost the activity. In 1937 economist John R. Hicks proposed a model that synthesizes classical theory and Keynesian theory, namely the IS-LM model.

The IS-LM model is composed of the two equations below: (i) the equilibrium equation in the goods and services market (IS) equals the equality between investment (I) and savings (S); (ii) the equilibrium equation in the money market (LM). In these equations, we have the interest rate (noted r), household consumption (noted C), investment noted I, public expenditure noted G, net exports noted NX, the money supply noted M , the level of prices noted p, the real GDP at the market price noted Y, the income taxes noted T.

$$Y = C(Y - T) + I(r) + G + NX \qquad \text{(IS)}$$

$$\frac{M}{p} = L(r, Y) \qquad \text{(LM)}$$

Graph 4: Effect of Expansionary Monetary and Budgetary Policies



In graph 4, the curve (IS) summarizes the relationship between the interest rate and real GDP from the goods and services market for a given fiscal policy. Any change in fiscal policy that increases demand for goods and services moves the curve (IS) to the right, for example from (IS) $_0$ to (IS) $_1$ in graph 4. Similarly, the curve (LM) synthesizes the relationship between the interest rate and real

money-market GDP, for a given level of supply of real money balances. Any increase in the supply of real cash balances causes the curve (LM) to move downwards, for example from $(LM)_0$ to $(LM)_1$.

According to the IS-LM model, if at time t_0, the economy is in a situation of underemployment with a real GDP level corresponding to Y_0, the implementation of a fiscal policy and an Expansionary monetary policy would lead to an increase in real GDP, for example from Y_0 to Y_1 as shown in graph 4, and thus underemployment would decline. However, the IS-LM model does not capture how real GDP growth leads to reduced unemployment. It is implicitly assumed that any economic growth induces a reduction in unemployment. Thus, while the IS-LM model remains an important tool for analyzing the effects of economic policies on economic growth, its use as a tool for analyzing the effects of economic policies on underemployment or unemployment is undermined by the fact that the link "growth-unemployment" remains his "black box". Hence the interest of integrating Okun's relationship with the IS-LM model in order to arrive at a better tool for analyzing the effects of economic policies on unemployment.

2. Integrated Model IS-LM-LO

To integrate the IS-LM model and Okun's law, we will use the relationship between the rate of real GDP growth (Y) between t_0 and t and the level of real GDP (Y) at t. Let us note the inverse of the real GDP at t_0 ($a_0 = 1 / Y_0$), then the growth rate g (Y) of real GDP between t_0 and t, i.e. ($\Delta Y / Y_0$), can be written under the form of the equation (RP) below.

$$g(Y) = a_0 Y - 1 \qquad \text{(RP)}$$

Then all the equations of the IS-LM-LO integrated model are as follows:

$$\begin{cases} Y = C(Y-T) + I(r) + G + NX & (IS) \\ \dfrac{M}{p} = L(r, Y) & (LM) \\ \Delta U = \beta(\lambda, \alpha)\left[g(Y) - g^*(\lambda, \alpha, \theta, d)\right] & (LO) \\ g(Y) = a_0 Y - 1 & (RP) \end{cases}$$

Thus, the IS-LM-LO integrated model includes four curves: the IS curve, the LM curve, the LO curve of Okun's relationship and the RP curve of the transit relation of the IS-LM model to the law of Okun. The IS curve moves under the effect of exogenous shocks on the demand for goods and services, the LM curve shifts under the effect of exogenous shocks on the money supply, the LO curve moves under the effect of exogenous shocks on average real wages, labor, import duties and net taxes on goods and services, real gross profits.

In a country, several events can be the cause of various exogenous shocks on the LO curve. For example, the entry into the mining phase of large deposits of mineral resources generally constitutes a significant exogenous shock both on average real wages and on real gross profits. Indeed, the wages and the gross surpluses of the mining industry being strongly raised compared to the rest of the economy, the average wage and the gross profit of the economy will increase strongly with the acceleration of the mining production. This will cause a shift in the LO curve. Also, the exceptional migratory phenomena of the labor force between the countries, the armed conflicts, as well as the important changes in the direction of the public expenditure and the policies of investments cause movements of the LO curve.

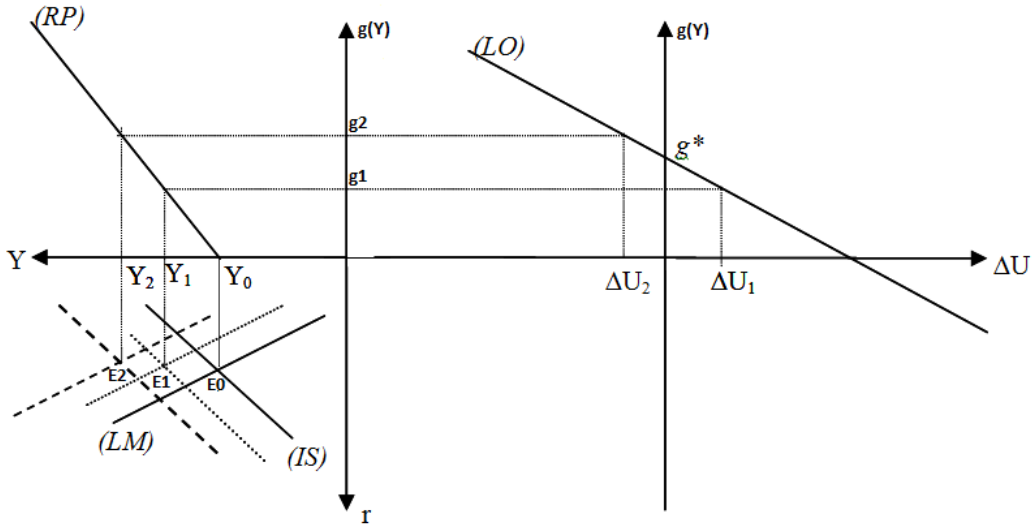
5. Analysis of the effects of economic policies on unemployment

In the 1930s to 1960s, Keynesian policies helped to reduce unemployment. However, since the 1970s the results of these policies have been rather mixed. Also, over the last two decades (1995-2015), many countries in Africa have experienced relatively high economic growth, but this has been accompanied by an increase in youth unemployment with persisting poverty. Why can economic policies or shocks to economic growth not reduce unemployment?

To answer this question, we rely on the integrated model IS-LM-LO. Three situations will be distinguished according to the nature of the changes in Okun's relationship induced by economic policies: (i) economic policies do not induce a change in Okun's relationship; (ii) economic policies induce an increase in g*; (iii) economic policies lead to a decrease in g*.

1. Effects of economic policies without changes in Okun's relationship

Graph 5: Effects of Shocks Not Inducing Changes in Okun's Relationship



Source: This Paper.

Let us consider that the economy is at point E_0 in a situation of underemployment equilibrium, as is the case nowadays in the majority of African countries. Then, the situation of the economy at the date t_0 is characterized by the curves IS and LM intersecting at the point E_0, the curve LO of the relationship of Okun and the "curve of passage" RP whose Y_0 is the abscissa at the origin (Graph 5). Suppose that the policy implemented is conducive to economic growth, but does not induce changes in Okun's relationship. That is to say, economic policy, although it helped to revive the economy, did not lead to an acceleration or deceleration in the growth of the average real wage, gross profits, rights to import and net taxes on goods and services; also, the demographic dynamics remain unchanged. In such a situation, two cases are possible depending on the magnitude of the shock caused by the economic policy.

In the first case, it is assumed that the shock is relatively small in magnitude. It moves the IS and LM curves to reach the equilibrium point E_1 (graph 5). As a result, real GDP grows from Y_0 to Y_1. Through the RP curve, we deduce that this increase in real GDP corresponds to a positive growth rate g1 which remains below the threshold g* of the Okun relationship. At growth level g_1, the change in the unemployment rate is positive and equal to ΔU_1. Thus, the economic growth induced by the state's recovery policy is not enough to reduce unemployment; it is accompanied by a rise in the unemployment rate.

In the second case, it is assumed that the magnitude of the shock is relatively high and moves the IS and LM curves so as to reach the equilibrium point E_2 (graph 5). As a result, real GDP grows

from Y_0 to Y_2. Through the transition curve RP, it is deducted that this increase in real GDP corresponds to a positive growth rate g_2 which is greater than the threshold g* of the Okun relationship. At growth level g_2, the change in the unemployment rate is negative and equal to ΔU_2. Thus, the economic growth induced by the policy is sufficient to reduce unemployment; it is accompanied by a fall in the unemployment rate.

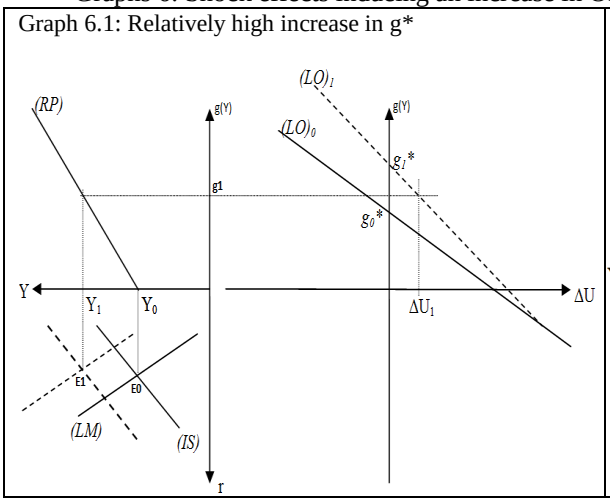
So, for an economic policy neutral towards Okun's relationship, the higher the rate of economic growth it induces is compared to Okun's growth threshold, the more the reduction of unemployment rate is important.

*2. Effects of economic policies leading to an increase in the Okun threshold g**

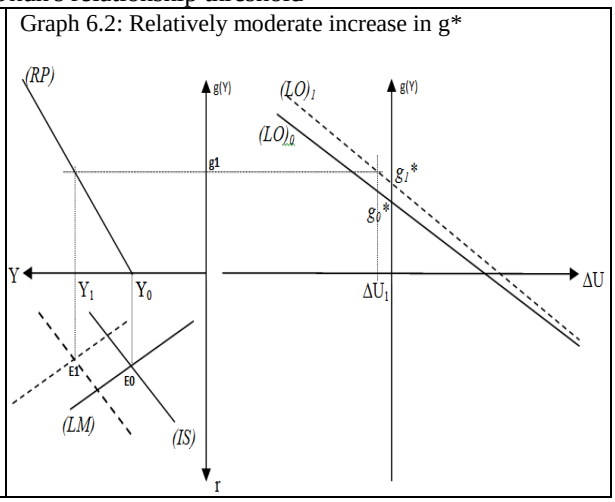
Let's consider that the economy is in a situation of underemployment at point E_0 with a real GDP level Y_0. The state implements a policy conducive to economic growth that shifts the economy from equilibrium E_0 to equilibrium E_1 with a level of real GDP Y_1 (graphs 6), with a growth rate economic g_1 greater than the threshold g_0* of the initial relationship of Okun materialized by the curve $(LO)_0$. Unlike the previous case, suppose that the stimulus policy induces a structural change in Okun's relationship so that the LO curve moves upwards as shown in both graphs 6. Okun's relationship now corresponds to curve $(LO)_1$ and has threshold g_1*. The change in the unemployment rate can be positive or negative depending on the extent of structural change in Okun's relationship.

Graphs 6: Shock effects inducing an increase in Okun's relationship threshold

Graph 6.1: Relatively high increase in g*



Graph 6.2: Relatively moderate increase in g*



Source: This Paper

If economic policy induces a significant structural change in Okun's relationship (graph 6.1) so that the new Okun g_1* threshold is greater than the growth rate g_1, then the change in the unemployment rate is positive. The sharp increase in the Okun threshold indicates that the economic growth induced by economic policy is characterized more by an acceleration of the increase in average real wages, real gross profits or net duties and taxes on goods and services, or that the shock was accompanied by an acceleration of the dynamics of the workforce following for example a massive migratory movement.

Conversely, if economic policy induces a structural change in Okun's relationship (graph 6.2) so that the new Okun g_1* threshold remains below the economic growth rate g_1, then the change in the unemployment rate is negative. However, the decline in the unemployment rate remains relatively low because of the structural change in Okun's relationship. So, in both cases, the structural change induced by economic policy in Okun's relationship is vicious.

In fact, some reforms to make the investment code more attractive for investors and economic policies based primarily on the promotion of capital-intensive sectors such as the mining sector, lead to an acceleration of the increase in gross profits and thus induce a vicious structural change in Okun's

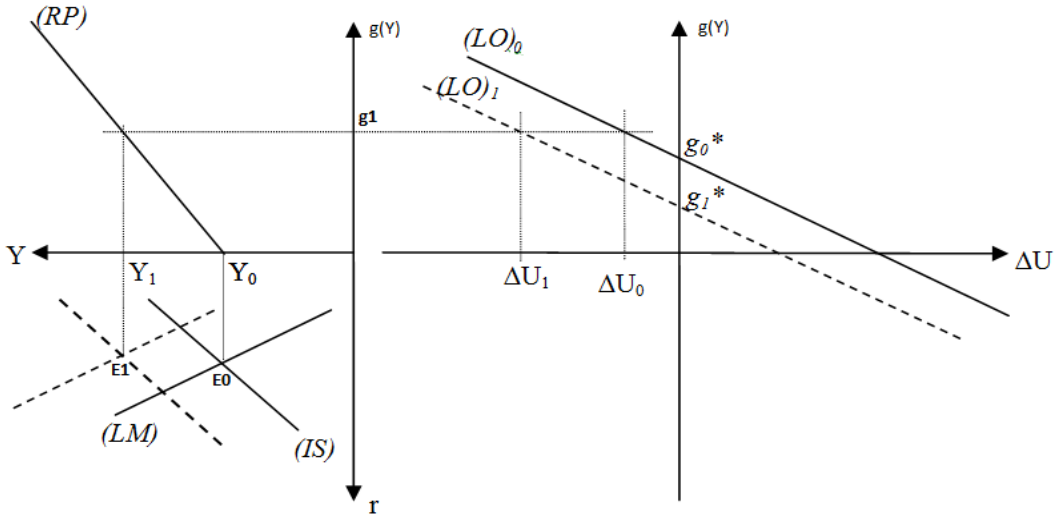
relationship. The same is true of expansionary wage policies and employment policies more favorable to skilled labor than to low or unskilled labor; because they accelerate the growth of the real average wage. Also, a fiscal policy funded by the increase in the door tax rate and/or the net tax on goods and services is accompanied by a vicious structural change in Okun's relationship.

Thus, when an economic recovery policy induces a vicious structural change in Okun's relationship, it reduces the unemployment rate only if its effect on the economic growth rate outweighs the structural change it induces in Okun's relationship. If not, it leads to an increase in the unemployment rate.

*3. Effects of economic policies leading to a decline in Okun threshold g**

Let's consider that the economy is in a state of underemployment equilibrium in point E_0 and that the state implements a policy to stimulate the economy and employment (graph 7). As a result of this policy, real GDP grows from Y_0 to Y_1, corresponding to an economic growth rate g_1 greater than g_0 *. As a result, with a fixed Okun relationship, the change in the unemployment rate should be negative and equal to ΔU_0. But, if the choice of expenditures made by the State as part of the economic stimulus induces a change in the Okun relationship so that the Okun threshold drops from g_0* to g_1*, the change in the unemployment rate would be ΔU_1, corresponding to a larger decline in the unemployment rate. So, we can say in this case that the structural change induced in Okun's relationship is virtuous.

Graph 7: Shock effects inducing a decline in the threshold of Okun's relationship



Source: This paper.

Economic recovery policies based on labor-intensive sectors such as the textile and manufacturing industries, as well as employment promotion policies targeting low or unskilled labor, are driving change in virtuous structural in Okun's relationship. Indeed, they cause a deceleration in the increase in gross profit and average real wages. The same is true for tax reforms aimed at reducing the burden of door taxation and net taxes on goods and services.

6. Conclusion

Starting from two universal relationships valid in any free enterprise economy, this paper has developed the theoretical foundations of the Okun law (LO) and an integrated model IS-LM-LO to better understand the effects of economic policies on unemployment.

As stated and demonstrated, the Okun Law Theorem confirms the existence of a negative relationship between the rate of economic growth and the change in the unemployment rate. However, this Okun relationship cannot be considered stable. In fact, it is determined on the one hand by the Okun coefficient β which decreases with the respective rates of growth of the active population and the average real wage, and on the other hand by the Okun threshold g^* which increases with the respective growth rates of labor force, average real wages, real gross profits, import duties and net taxes on goods and services. So, shocks on labor force, real average wage, real gross profits, or import duties and net taxes on goods and services, induce structural changes in Okun's relationship. The structural change in Okun's relationship can be seen in particular by a lowering of the Okun threshold g^*, in which case it is virtuous or conversely by a rise in the threshold, in which case it is vicious.

Because of the vicious structural changes that can occur in the Okun's relationship, economic growth, no matter how high it is, does not always lead to a reduction in unemployment. Indeed, when the economic dynamic creates a vicious structural change in Okun's relationship so that the new Okun threshold that it induces is always greater than the observed growth rate, the unemployment rate increases. An economic dynamic that engenders a vicious structural change in Okun's relationship reduces the unemployment rate only if the observed growth rate is greater than the new Okun threshold generated. Economic growth has a greater impact on unemployment when it is strong and engenders a virtuous structural change in Okun's relationship.

These results are evidence that the IS-LM model is insufficient to grasp the effects of economic policies on unemployment because, according to Okun's Law theorem, economic recovery cannot be systematically equated with a reduction in unemployment. The integration of Okun's relationship with the IS-LM model provides a more appropriate tool for analyzing the effects of economic policies on unemployment. From the analysis of this new integrated model IS-LM-LO, it is obvious that economic policy (fiscal and/or monetary), although having a positive effect on economic growth, would lead to an increase in unemployment if it creates a vicious structural change in Okun's relationship. The most effective economic stimulus in the fight against unemployment is one that, in addition to accelerating economic growth, induces a virtuous structural change in Okun's relationship.

Therefore, the theorem of Okun's law opens new perspectives, in particular empirically, on the one hand the analysis of the quality of the economic growth and on the other hand for the objective evaluation of the quality of the economic policy options from the perspective of their impact on employment.

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